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APPLICATION OF THE GAMMA FUNCTION

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Annotation

This article explores Euler integrals, specifically the gamma and beta functions, and their significance in mathematical analysis, with applications in differential equations, probability theory, and statistics. The gamma function, defined as a continuous extension of the factorial, is analyzed for its key properties, including continuity, infinite differentiability, and the functional relation $\Gamma(z+1)=z\Gamma(z)$. Singular points ($t=0$ and $t=\infty$) and the uniform convergence of the integral are examined using the Weierstrass criterion. Applications of the gamma function in probability distributions, quantum mechanics, and statistical physics are highlighted through examples. The article also includes a practical integral calculation and is supported by references to mathematical literature.

Keywords: Gamma function, Euler integrals, beta function, probability theory, statistics, differential equations.

Euler Integrals.

Euler integrals are among the fundamental mathematical tools that include gamma and beta functions. Both functions represent continuous functions and special processes, and are widely applied in the fields of differential equations, probability theory, and statistics.

Gamma Function

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The gamma function is particularly widely used in probability distributions, such as the gamma distribution. This function is a continuous equivalent of the factorial and is employed in numerous statistical models. For instance, in the process of multidimensional statistical modeling, distributions and integral calculations are generated using the gamma function. In physics, the gamma function also serves as an essential tool for models in quantum mechanics and statistical physics. The gamma function is a function that generalizes factorials and is defined as follows:

Γ

z The points t=0 and t=∞ are called singular points. Let's express the integral (1) as the sum of two integrals: $t^{z-1}e^{-t}dt$, Re(z)>0 (1)

Γ

z These two integrals are uniformly convergent in any finite interval with respect to the parameter according to the Weierstrass criterion. Indeed, if: $t^{z-1}e^{-t}dt + 1 \; t^{z-1}e^{-t}dt$

Properties of the gamma function.

. (1) The gamma function is continuous for all values of the argument and has continuous derivatives of all orders. By differentiating the integral (1) under the integral sign,

$\Gamma' = \int_1^{+\infty} t^{z-1} \ln t \; e^{-t} dt$ ni hosil qilamiz.

$\int_0^1 t^{z-1} \ln t \; e^{-t} dt$ va $\int_1^{+\infty} t^{z-1} \ln t \; e^{-t} dt$*Since the integrals are uniformly convergent over an arbitrary interval, it is possible to apply Leibniz's rule:*

For the first case, when t=0, the majorant function is [missing information], while for the second case, when [missing information], the majorant function is [missing information]. Based on induction, it can be shown that Γ (z) is an infinitely differentiable function for z>0.

$\Gamma^{(n)}(z) = \int_1^{+\infty} t^{z-1} (\ln t) e^{-t} dt$ (2)

2^0. Now let us derive the basic functional relation for the Gamma function. Let . As a result of integration by parts, the following

$$\Gamma(z+1) = \int_1^{+\infty} t^z e^{-t} dt = -e^t t^z \;\Big|$$

Γ(z+1) = z Γ(z) (3)

ni hosil qilamiz. (18) formula *pasaytirish formulasi* deyiladi. Bu formuladan xususiy holda natural lar uchun

$$\Gamma(n+1) = n!$$

We take the formula. From formula (3), it is possible to determine the behavior of the function Γ (z) with respect to z: since Γ (z+1) is a continuous function and because it is

$$\Gamma(z) = \frac{\Gamma(z+1)}{z} \sim \frac{\Gamma(1)}{z}.$$

Formula (3) allows for the extension of the Γ (z) function, while preserving its properties, even for negative values of z that are not equal to -1, -2, -3, ..., -n, ...

$$\Gamma(z) = \frac{\Gamma(z+1)}{z} \quad (4)$$

will be. Since -1<z<0, the definition (4) is correct. Let's consider the character of the function at y=z+1. Taking y=z+1, we obtain an equivalent function to y=z+1. Therefore, instead of (4) in

$$\Gamma(y-1) = \frac{\Gamma(y)}{y-1} \sim -\Gamma(y) \sim -\frac{1}{y} = -\frac{1}{z+1};$$

Let's consider this. Therefore, $z \to -1+0$ da $\Gamma(z) \sim -\frac{1}{z+1}$, mainly $z \to -n$ da $\Gamma(z) \sim -\frac{(-1)^n}{z+n}$ it can be determined..[1-15]

Example, $\int_0^{+\infty} x^{2n} e^{-x^2}\,dx$, ³ $n \in N$, calculate the integral.

Solution: $x \longrightarrow \sqrt{t}$, (t>0)saying that,

$$\int_0^{+\infty} x^{2n} e^{-x^2}\,dx = \frac{1}{2}\int_0^{+\infty} t^{n-\frac{1}{2}} e^{-t}\,dt = \frac{1}{2}\Gamma(n+2) = \frac{(2n+1)!}{2^{n+1}}\pi;$$

we produce..

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