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WITH A SMALL PARAMETER IN FRONT OF A HIGHER ORDER DERIVATIVE

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ANNOTATION

This in the article high orderly derivative in front of small parameter participated differential equations solution for initially integration method offer The method is main idea – in the equation the most high orderly derivative Chebyshev polynomials in a row spread , then him/her two times integrate and solve row in appearance is to express . Example as a rule second orderly linear equation $\varepsilon \frac{d^2u}{dy^2} + \frac{1}{2}\frac{du}{dy} = \frac{1}{8}(y+1), \quad y \in (-1,1),$ borderline conditions with considered . Clearly solution with comparison for analytical expression The method is given below . other high orderly apply to equations as well possible .

Key words . Small parameter , High orderly derivative , Chebyshev plurals , originally integration method , boundary value problem, singular differential equations , exact solution and approximate analytical method

ENTRANCE

Small parametric differential equations physics , mechanics and other engineering in the fields borderline layer , turbulent flows and relaxation processes in modeling wide occurs . Such in equations the most high orderly derivative in front of small parameter because of solution sharp to a variable (singular) property has will be . Definitely the solution find many in cases difficult or possible not , that's why for approximate analytical and numerical methods working Traditional methods (e.g. , asymptotic extensions , separate schemes) some low accuracy in cases to give possible .

This in the article author initially integration method offer The method essence – in the equation the most high orderly derivative Chebyshev many things according to in line spread , then this derivative necessary times integrate , solution himself and lower orderly derivatives rows in appearance to express . Integration immutable borderline from the conditions This approach is defined as spectral of methods one type to be , to be high accuracy and fast to approach provides . In the article of the method main stages clear example through explained and comparison for of the matter analytical solution cited .

Initially integration method high orderly derivative in front of small parameter was equation to solve implementation following differential equation in the example of considered is , it is other high orderly easily to equations generalization possible

$$\varepsilon\frac{d^2u}{dy^2}+\frac{1}{2}\frac{du}{dy}=\frac{1}{8}(y+1),\qquad y\in(-1,1), \tag{1}$$

this borderline conditions with considered

$$u(-1)=u(1)=0, \tag{2}$$

this on the ground ε - small parameter . The differential problem (2.1) - (2.2) of clear solution

$$u(y)=\frac{\varepsilon-0.5}{1-\varepsilon^{-1/\varepsilon}}\left(1-\varepsilon^{-(y+1)/2\varepsilon}\right)-\varepsilon\frac{y+1}{2}+\frac{(y+1)^2}{8} \tag{3}$$

to look has . This solution of the matter initially integration method with taken approximate solution with comparison for necessary will be . In equation (1) high orderly derivative and right on the side $f(y)$ function below limited rows in appearance is searched for :

$$\frac{d^2u}{dy^2}=\sum_{i=0}^{N}a_iT_i(y),\qquad f(y)=\sum_{i=0}^{N}b_iT_i(y), \tag{4}$$

this on the ground $T_i(y)$ - i - orderly first round Chebyshev polynomials , a_i, b_i - unknown coefficients . Initially integration method main in the idea of formula (4) high orderly until the derivative problem is solved until initially integrally to take in mind is captured . For this purpose , in line (4) derivative initially two times integrated , the following expressions harvest we do :

$$\frac{du}{dy} = \sum_{j=0}^{N+1}\sum_{i=0}^{N} f_{ji}^{(1)} a_i T_j(y) + C_1 T_0(y), \tag{5}$$

$$u(y) = \sum_{j=0}^{N+2}\sum_{i=0}^{N} f_{ji}^{(0)} a_i T_j(y) + C_1 T_1(y) + C_2 T_0(y), \tag{6}$$

there, C_2 - C_1 unknown integration Invariants . To determine them, the boundary conditions (2) are used. and we use the following properties of Chebyshev polynomials: $T_n(\pm 1) = (\pm 1)^n$

In this case following to equality has we will be :

$$u(+1) = \sum_{j=0}^{N+2}\sum_{i=0}^{N} f_{ji}^{(0)} a_i + C_1 + C_2 = 0, \tag{7}$$

$$u(-1) = \sum_{j=0}^{N+2}\sum_{i=0}^{N} (-1)^j f_{ji}^{(0)} a_i - C_1 + C_2 = 0. \tag{8}$$

Unchanged C $_2$ what determination for equations (7) and (8) initially we add :

$$u(+1) + u(-1) = \sum_{j=0}^{N+2}\sum_{i=0}^{N} f_{ji}^{(0)} a_i + \sum_{j=0}^{N+2}\sum_{i=0}^{N} (-1)^j f_{ji}^{(0)} a_i + 2C_2 = 0,$$

this from the equation C $_2$ is the constant clearly we get :

$$C_2 = -\frac{1}{2}\sum_{i=0}^{N}\left[\sum_{j=0}^{N+2} f_{ji}^{(0)} + (-1)^j f_{ji}^{(0)}\right] a_i .$$

Same also the constant C $_1$ determination for Equation (7) to (8) minus , C $_1$ unchanging for following expression harvest we will do

$$C_1 = \frac{1}{2}\sum_{i=0}^{N}\left[\sum_{j=0}^{N+2}\left((-1)^j f_{ji}^{(0)} - \sum_{j=0}^{N+2} f_{ji}^{(0)}\right)\right]a_i \ .$$

The following designations input through

$$\delta_i^{(0)} = \sum_{j=0}^{N+2} f_{ji}^{(0)}, \qquad \bar{\delta}_i^{(0)} = \sum_{j=0}^{N+2}(-1)^j f_{ji}^{(0)},$$

C $_1$ and C $_2$ immutable for expressions this in appearance writing we get :

$$C_1 = \frac{1}{2}\sum_{i=0}^{N}\left[\bar{\delta}_i^{(0)} + \delta_i^{(0)}\right]a_i \qquad (9)$$

$$C_2 = -\frac{1}{2}\sum_{i=0}^{N}\left[\delta_i^{(0)} + \bar{\delta}_i^{(0)}\right]a_i \qquad (10)$$

The constants (9), (10) in consideration received without formulas (5), (6) following general in the form to write possible :

$$u^{(\beta)}(y) = \sum_{j=0}^{N+2-\beta}\sum_{i=0}^{N} g_{ji}^{(\beta)} a_i T_j(y), \ \ \beta = 0,1, \qquad (11)$$

this on the ground

$$g_{ji}^{(1)} = f_{ji}^{(1)} + \delta_{j,0}\frac{1}{2}\left(\bar{\delta}_i^{(0)} - \delta_i^{(0)}\right), \qquad (12)$$

$$g_{ji}^{(0)} = f_{ji}^{(0)} + \delta_{j,1}\frac{1}{2}\left(\bar{\delta}_i^{(0)} - \delta_i^{(0)}\right) - \delta_{j,0}\frac{1}{2}\left(\delta_i^{(0)} - \bar{\delta}_i^{(0)}\right), \qquad (13)$$

in this

$$\delta_{ij} = \begin{cases} 1, agar\ i = j, \\ 0, agar\, i \neq j, \end{cases} - \text{Kronecker symbol}$$

Now , lines (4), (11) are replaced by (1) put it down , one kind orderly polynomials in front of coefficients equalize , unknown odds $a_{0,} a_1 a_2 ..., a_N$ what determination for following linear algebraic to the system has we will be :

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$$\sum_{k=0}^{N}\left[\varepsilon\delta_{ik}+\frac{1}{2}g_{ik}^{(1)}\right]a_k=b_i,\qquad i=0,1,2,\ldots,N \tag{14}$$

System (14) of right side as follows is determined ,

as is known ,

$$f(y)=\frac{1}{8}(y+1)=\sum_{i=0}^{N}b_iT_i(y), \tag{15}$$

from this in formula (15) b_i coefficients , as follows reverse replacement with is determined

$$b_i=\frac{2}{Nc_i}\sum_{l=0}^{N}\frac{1}{c_l}f(y_l)T_i(y_l),\qquad i=0,1,\ldots,N$$

or

$$b_i=\frac{1}{4Nc_i}\sum_{l=0}^{N}\frac{1}{c_l}(y_l+1)T_i(y_l),\qquad i=0,1,\ldots,N \tag{16}$$

this is on the ground $c_0=c_N=2,\ c_l=1,$ if $l\neq 0;N$, $y_l=\cos\frac{\pi l}{N}$ - first kind of Chebyshev polynomials collocation nodes .

In formulas (11) - (13) the immutable calculation algorithm we bring :

$$\delta_i^{(\beta)}=\sum_{j=0}^{N+2-\beta}f_{ji}^{(\beta)},\qquad \bar{\delta}_i^{(\beta)}=\sum_{j=0}^{N+2-\beta}(-1)^j f_{ji}^{(\beta)},\ \beta=0;1$$

$$f_{ji}^{(1)}=\delta_{j,i+1}\beta_i^{(1)}+\delta_{j,i-1}\zeta_i^{(1)},$$
$$f_{ji}^{(0)}=\delta_{j,i+2}\beta_i^{(0)}+\delta_{j,i}\zeta_i^{(0)}+\delta_{j,i-2}\mu_i^{(0)},$$

this they are immutable on earth β,ζ,μ following formulas through is :

$$\beta_i^{(1)} = \frac{c_i}{2(i+1)},\ i \ge a,\quad \beta_i^{(0)} = \frac{\beta_i^{(1)}}{2(i+2)},\quad i \ge 0,$$

$$\zeta_i^{(1)} = \frac{-1}{2(i-1)},\ i \ge 2,\quad \zeta_i^{(0)} = \frac{\zeta_i^{(1)} - \beta_i^{(1)}}{2i},\quad i \ge 1,$$

$$\mu_i^0 = \frac{-\zeta_i^{(1)}}{2(i-2)},\ i \ge 3.$$

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