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NUMERICAL MODELING OF NONLINEAR PARABOLIC TYPE DIFFERENTIAL EQUATION

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ANNOTATION

In this article, the boundary value problem posed for a one-dimensional quasilinear heat transfer equation of a nonlinear parabolic type is numerically modeled using an explicit separation scheme. The heat transfer coefficient is taken as proportional to temperature: $k(u)=u$ The initial temperature distribution is parabolic, and the temperature at the boundaries is assumed to be zero. The stability condition of the separation scheme obtained as a result of discretization is determined and an example of calculation for one time step is given. The proposed method is simple and easy to program, and can be used in modeling nonlinear heat transfer processes.

Keywords: nonlinear parabolic equation, quasilinear heat transfer, explicit separation scheme, Dirichlet boundary condition, numerical modeling.

CHIZIQLI BO'LMAGAN PARABOLIK TIPDAGI DIFFERENTSIAL TENGLAMANI SONLI MODELLASHTIRISH

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ANNOTATSIYA

Ushbu maqolada chiziqli bo'lmagan parabolik tipdagi differentsial tenglama bir o'lchamli kvazichiziqli issiqlik o'tkazuvchanlik tenglamasi uchun qo'yilgan chegaraviy masala oshkor (explicit) ayirma sxemasi yordamida sonli modellashtirilgan. Issiqlik o'tkazuvchanlik koeffitsiyenti temperaturaga proporsional deb olingan: $k(u) = u$ Boshlang'ich temperatura taqsimoti parabolik shaklda, chegaralarda temperatura nolga teng deb qabul qilingan. Diskretlashtirish natijasida olingan ayirma sxemasining barqarorlik sharti aniqlangan va bir vaqt qadami uchun hisoblash misoli keltirilgan. Taklif etilgan usul sodda va dasturlashga qulay bo'lib, nochiziqli issiqlik tarqalish jarayonlarini modellashtirishda qo'llanilishi mumkin.

Kalit so'zlar: chiziqli bo'lmagan parabolik tenglama, kvazichiziqli issiqlik o'tkazuvchanlik, oshkor ayirma sxemasi, Dirixle chegaraviy sharti, sonli modellashtirish.

Kirish

Ko'pgina real fizik jarayonlar (issiqlik tarqalishi, diffuziya, filtratsiya va boshqalar) parabolik tipdagi differentsial tenglamalar bilan tavsiflanadi. Agar jarayonning xarakteristikalari (masalan, issiqlik o'tkazuvchanlik koeffitsiyenti)

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o‘zgarmas bo‘lsa, tenglama chiziqli bo‘ladi. Biroq ko‘p hollarda muhit xossalari temperaturaga yoki konsentratsiyaga bog‘liq bo‘lib, bu chiziqli bo‘lmagan (xususan, kvazichiziqli) tenglamalarga olib keladi.

Chiziqli bo‘lmagan parabolik tenglamalarni analitik usullarda yechish faqat ayrim maxsus holatlarda mumkin. Shuning uchun sonli usullar muhim ahamiyat kasb etadi. Ushbu maqolada quyidagi ko‘rinishdagi bir o‘lchamli kvazichiziqli issiqlik o‘tkazuvchanlik tenglamasi qaraladi:

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(k(u)\frac{\partial u}{\partial x}\right), \; 0 < x < L, t > 0$$

bu yerda $u(x,t)-$ temperature, $k(u)-$issiqlik o‘tkazuvchanlik koeffitsiyenti (temperaturaga bog’liq).

Maqolada quyidagi aniq masala sonli modellashtiriladi:

Soha $L = 1m, T = 0{,}01s$ (yakuniy vaqt) issiqlik o‘tkazuvchanlik koeffitsienti: $k(u) = u$ (chiziqli bog`liqlik).

Boshlang’ich shart: $u(x,0) = 100x(1-x)$ (parabolik taqsimot,maksimal 25°C)

Chegaraviy shartlar: $u(0,t) = 0, \quad u(1,t) = 0$ (bir jinsli Dirixle).

Fizik talqini:uchlarida temperature nol darajada saqlanayotgan sterjendagi issiqlik tarqalishi.

To‘rni kiritamiz:

$$\varpi_{ih} = \begin{cases} x_i = ih, \; i = 0,1,2,...,N, \; h = \dfrac{L}{N}; \\ t_j = j\tau, \; j = 0,1,2,...,M, \; \tau = \dfrac{T}{M}. \end{cases}$$

izlanayotgan funksiyaning to‘rdagi qiymati: $u_i^j \approx \left(x_i, t_j\right)$

Vaqt bo`yicha oldinga farq, fazo bo‘yicha markaziy farqdan foydalanamiz:

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$$\frac{u_i^{j+1}-u_i^j}{\tau}=\frac{1}{h}\left[k\left(\frac{u_i^j+u_{i+1}^j}{2}\right)\frac{u_{i+1}^j-u_i^j}{h}-k\left(\frac{u_{i-1}^j+u_i^j}{2}\right)\frac{u_i^j-u_{i-1}^j}{h}\right].$$

$k(u)=u$ bo‘lganligi uchun $k\left(\frac{u_i^j+u_{i+1}^j}{2}\right)=\frac{u_i^j+u_{i+1}^j}{2}$ shuning uchun:

$$u_i^{j+1}=u_i^j+\frac{\tau}{h^2}\left[\frac{u_i^j+u_{i+1}^j}{2}(u_{i+1}^j-u_i^j)-\frac{u_{i-1}^j-u_{i-1}^j}{2}(u_i^j-u_{i-1}^j)\right].$$

$$u_i^{j+1}=u_i^j+\frac{\tau}{2h^2}\left[(u_{i+1}^j)^2-(u_i^j)^2-(u_i^j)+(u_{i-1}^j)^2\right]=u_i^j+\frac{\tau}{2h^2}\left[(u_{i+1}^j)-2(u_i^j)^2+(u_{i-1}^j)^2\right]$$

Shunday qilib, **Oshkor ayirma sxemasi**ning yakuniy ko`rinishi:

$$u_i^{j+1}=u_i^j+\frac{\tau}{2h^2}\left[(u_{i+1}^j)^2-2(u_i^j)^2+(u_{i-1}^j)^2\right],\ \ i=1,\ldots,N-1$$

Chegaraviy nuqtalar: $u_0^{j+1}=0,\ u_N^{j+1}=0.$

Turg’unlik sharti

Oshkor sxemalar shartli turg’un bo‘ladi. Chiziqli taxminda ($k=const$) turg’unlik sharti: EMBED Equation.3 $\tau\le h^2/(2k)$. Kvazichiziqli holda k o‘rniga $k(u)$ ning maksimal qiy

$$\tau\le\frac{h^2}{2\cdot 25}=\frac{h^2}{50}.$$

Agar $h=0.1$ tanlansa, $\tau\le 0.0002$. Hisoblashlarda $\tau=0.0001$ deb olinadi.Bu shartni qanoatlantiradi va sxemaning turg’unlik ishlashini ta’minlaydi.

Hisoblash misoli (birinchi vaqt qadami) $N=10(h=0.1),\ \tau=0.0001$ bo‘lsin.

Boshlang‘ich qatlam (n=0):

$$u_i^0=100x_i(1-x_i),\ i=0,\ldots,10.$$

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i	x_i	u_i^0
0	0.0	0.0
1	0.1	9.0
2	0.2	16.0
3	0.3	21.0
4	0.4	24.0
5	0.5	25.0
6	0.6	24.0
7	0.7	21.0
8	0.8	16.0
9	0.9	9.0
10	1.0	0.0

$$\frac{\tau}{2h^2} = \frac{0.0001}{2 \cdot 0.01} = 0.005$$

$$u_1^1 = 9.0 + 0.005(16^2 - 2 \cdot 9^2 + 0^2) = 9.0 + 0.005(256 - 162) = 9.0 + 0.47 = 9.47$$

u_i^1

$$u_2^1 = 16.0 + 0.005(21^2 - 2 \cdot 16^2 + 9^2) = 16.0 + 0.005(441 - 512 + 81) = 16.0 + 0.05 = 16.05$$

$$u_3^1 = 21.0 + 0.005(24^2 - 2 \cdot 21^2 + 16^2) = 21.0 + 0.005(576 - 882 + 256) = 21.0 - 0.25 = 20.75$$

$$u_4^1 = 24.0 + 0.005(25^2 - 2 \cdot 24^2 + 21^2) = 24.0 + 0.005(625 - 1152 + 441) = 24.0 - 0.43 = 23.57$$

$$u_5^1 = 25.0 + 0.005(24^2 - 2 \cdot 25^2 + 24^2) = 25.0 + 0.005(576 - 1250 + 576) = 25.0 - 0.49 = 24.51$$

$$u_6^1 = u_4^1 = 23.57, \quad u_7^1 = u_3^1 = 20.75, \quad u_8^1 = u_2^1 = 16.05, \quad u_9^1 = u_1^1 = 9.47, \quad u_0^1 = u_{10}^1 = 0$$

i	x_i	u_i^0	u_i^1
0	0.0	0.0	0.00
1	0.1	9.0	9.47
2	0.2	16.0	16.05
3	0.3	21.0	20.75
4	0.4	24.0	23.57
5	0.5	25.0	24.51
6	0.6	24.0	23.57
7	0.7	21.0	20.75
8	0.8	16.0	16.05

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9	0.9	9.0	9.47
10	1.0	0.0	0.00

Markaziy temperatura 25.0 dan 24.51 ga pasaygan,unga yaqin nuqtalarda

Keyingi vaqt qadamlarida xuddi shu formula takrorlanadi. Har bir qadamda temperature maydoni silliq ravishda nolga intiladi.

Natijalarni tahlil qilish

Oshkor sxema quyidagi afzalliklarga ega

sodda va aniq-har bir yangi qatlam qiymati to`g`ridan to`g`ri oldingi qatlam orqali hisoblanadi.

Dasturlashga qulay-iterativ skillarni talab qilmaydi.

Nochiziqli tenglamalar uchun tabiiy-nochiziqli oshkor holda to‘liq saqlanadi

T

Uzoq vaqt oralig’i (T katta) uchun ko‘p qadamlar talab etiladi.

Xulosa

Ushbu maqolada chiziqli bo‘lmagan parabolik tipdagi differensial tenglama bir

kichik bo‘lishi kerak; bu esa katta hisoblashlarga olib kelishi mumkin.

o‘lchamli kvazichiziqli issiqlik o‘tkazuvchanlik tenglamasi uchun qo‘yilgan masala oshkor ayirmali sxemasi yordamida sonli mosellashtirildi.

Oshkor sxemaning aniq ko‘rinishi keltirildi.

Turg’unlik sharti $\tau \le h^2/(2\max k(u))$ ekanligi ko‘rsatildi.

Bir vaqt qadami uchun hisoblash misoli batafsil bayon qilindi.

Taklif etilgan usul talimiy maqsadlarda va soda fizik jarayonlarni modellashtirishda samarali qo‘llash mumkin. Kelgusida ushbu sxemani o‘zgaruvchan $k(u)$ (masalan, $k(u)=u$ yoki $k(u)=1+u$ va ikki o‘lchamli holatlarga umumlashtirish rejalashtirilgan.

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